

Gravitational Wave in Media

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KMI film: https://www.youtube.com/watch?v=TP2WFsN_k6w

S. Capozziello, M. De Laurentis, S. Nojiri and S. D. Odintsov,
“Evolution of gravitons in accelerating cosmologies: The case of extended gravity,”
Phys. Rev. D **95** (2017) no.8, 083524
doi:10.1103/PhysRevD.95.083524
arXiv:1702.05517 [gr-qc]

S. Nojiri and S. D. Odintsov,
“Cosmological Bound from the Neutron Star Merger GW170817 in scalar-tensor and $F(R)$ gravity theories,”
Phys. Lett. B **779** (2018) 425
doi:10.1016/j.physletb.2018.01.078
arXiv:1711.00492 [astro-ph.CO].

K. Bamba, S. Nojiri and S. D. Odintsov,
“Propagation of gravitational waves in strong magnetic fields,”
Phys. Rev. D **98** (2018) no.2, 024002
doi:10.1103/PhysRevD.98.024002
arXiv:1804.02275 [gr-qc].

I. Brevik, S. Nojiri and S. D. Odintsov, in preparation.

Introduction

Gravitational Waves

$\Leftarrow$ linearizing ($g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}$) the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2 T_{\mu\nu}$$

by choosing the transverse and traceless gauge,

$$\nabla^\mu h_{\mu\nu} = g^{\mu\nu} h_{\mu\nu} = 0$$

$\Rightarrow$

$$\begin{aligned} & \frac{1}{2} \left[-\nabla^2 h_{\mu\nu} - 2R^\lambda{}_\nu{}^\rho{}_\mu h_{\lambda\rho} + R^\rho{}_\mu h_{\rho\nu} + R^\rho{}_\nu h_{\rho\mu} - h_{\mu\nu} R + g_{\mu\nu} R^{\rho\lambda} h_{\rho\lambda} \right] \\ & = \kappa^2 \delta T_{\mu\nu}^{\text{tt}} \text{ (transverse and traceless part).} \end{aligned}$$

$T_{\mu\nu}$ depends on the metric.

The dependence carries the informations on the mechanism of the expansion of the universe.

$\delta T_{\mu\nu}^{\text{tt}}$ can be different in models even if the expansion history of the universe is identical.

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Example: Scalar Tensor Theory

$$S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\phi, \quad \mathcal{L}_\phi = -\frac{1}{2} \omega(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi),$$

$$\Rightarrow T_{\mu\nu} = -\omega(\phi) \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} \mathcal{L}_\phi,$$

$$\Rightarrow \delta T_{\mu\nu} = h_{\mu\nu} \mathcal{L}_\phi + \frac{1}{2} g_{\mu\nu} \omega(\phi) \partial^\rho \phi \partial^\lambda \phi h_{\rho\lambda},$$

Assuming a FRW spatially flat metric

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2,$$

and $\phi = \phi(t)$, we may choose $\phi = t$

$$\left(\phi = \phi(\tilde{\phi}) \Rightarrow \omega(\phi) \partial_\mu \phi \partial^\mu \phi = \tilde{\omega}(\tilde{\phi}) \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi}, \quad \tilde{\omega}(\tilde{\phi}) \equiv \omega(\phi(\tilde{\phi})) \phi'(\tilde{\phi})^2 \right),$$

the FRW equations ($H \equiv \frac{\dot{a}}{a}$)

$$\begin{aligned} \frac{3}{\kappa^2} H^2 &= \frac{\omega}{2} + V, & -\frac{1}{\kappa^2} (2\dot{H} + 3H^2) &= \frac{\omega}{2} - V, \\ \Rightarrow \omega &= -\frac{2}{\kappa^2} \dot{H}, & V &= \frac{1}{\kappa^2} (\dot{H} + 3H^2). \end{aligned}$$

Then we can construct a model which reproduces any given history of expansion by $\omega(t \rightarrow \phi)$, $V(t \rightarrow \phi)$.

$$\text{ex. } a(t) = \left(\frac{t}{t_0} \right)^\alpha \Leftrightarrow \omega(\phi) = \frac{2\alpha}{\kappa^2 t_0^2 \phi^2}, \quad V(\phi) = \frac{3\alpha^2 - \alpha}{\kappa^2 t_0^2 \phi^2}.$$

t_0, α : real constants

$$\Leftrightarrow \alpha = \frac{2}{3(1+w)}.$$

w : equation of state (EoS) parameter
(when Universe is filled with perfect fluid with $w = p/\rho$).

$w = 0 \Leftrightarrow$ dust $\sim$ cold dark matter (CDM)

$$\omega(\phi) = \frac{4}{3\kappa^2 t_0^2 \phi^2}, \quad V(\phi) = \frac{2}{3\kappa^2 t_0^2 \phi^2}.$$

$w = \frac{1}{3} \Leftrightarrow$ radiation

$$\omega(\phi) = \frac{1}{\kappa^2 t_0^2 \phi^2}, \quad V(\phi) = \frac{1}{4\kappa^2 t_0^2 \phi^2}.$$

$$\frac{\partial T_{\mu\nu}}{\partial g_{\rho\sigma}} = \frac{1}{2} (\delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} + \delta_{\mu}^{\sigma} \delta_{\nu}^{\rho}) \left(-\frac{1}{2} g^{\eta\zeta} \omega(\phi) \partial_{\eta} \phi \partial_{\zeta} \phi - V(\phi) \right) + \frac{1}{2} g_{\mu\nu} \omega(\phi) \partial^{\rho} \phi \partial^{\sigma} \phi.$$

In FRW universe, $\phi = \phi(t)$,

$$\frac{\partial T_{ij}}{\partial g_{kl}} = \frac{1}{2} (\delta_i^k \delta_j^l + \delta_i^l \delta_j^k) \left(\frac{1}{2} \dot{\phi}^2 - V(\phi) \right).$$

For dust ($w = 0$),

$$p = \frac{1}{2} \dot{\phi}^2 - V(\phi) = 0 \Rightarrow \frac{\partial T_{ij}}{\partial g_{kl}} = 0,$$

For radiation ($w = \frac{1}{3}$)

$$\frac{\partial T_{ij}}{\partial g_{kl}} = \frac{1}{2} (\delta_i^k \delta_j^l + \delta_i^l \delta_j^k) \frac{1}{4\kappa^2 t_0^2 \phi^2},$$

Example: Quantum Thermodynamical Scalar Field

Free real scalar field ϕ with mass M in the flat background,

$$T_{00} = \rho = \frac{1}{2} \left(\pi^2 + \sum_{n=1,2,3} (\partial_n \phi)^2 + M^2 \phi^2 \right),$$

$$T_{ij} = \partial_i \phi \partial_j \phi + \frac{1}{2} \delta_{ij} \left(\pi^2 - \sum_{n=1,2,3} (\partial_n \phi)^2 - M^2 \phi^2 \right).$$

$\pi = \dot{\phi}$: momentum conjugate to ϕ .

$$\frac{\partial T_{ij}}{\partial g_{kl}} = \frac{1}{4} \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k \right) \left(\pi^2 - \sum_{n=1,2,3} (\partial_n \phi)^2 - M^2 \phi^2 \right) + \frac{1}{2} \delta_{ij} \partial^k \phi \partial^l \phi.$$

At Finite Temperature T .

$$\left\langle : \frac{\partial T_{ij}}{\partial g_{kl}} : \right\rangle_T = \frac{1}{12\pi^2} \delta_{ij} \delta^{kl} \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + M^2}} \frac{e^{-\beta(k^2 + M^2)^{\frac{1}{2}} - i\mu}}{1 - e^{-\beta(k^2 + M^2)^{\frac{1}{2}} - i\mu}},$$

$$M \rightarrow 0 \Rightarrow \left\langle : \frac{\partial T_{ij}}{\partial g_{kl}} : \right\rangle_{T, M=0} = \frac{1}{12\pi^2} \delta_{ij} \delta^{kl} \int_0^\infty dk \frac{k^3 e^{-\beta k - i\mu}}{1 - e^{-\beta k - i\mu}}.$$

Number density n :

$$\langle n \rangle_{T, M=0} = \frac{1}{2\pi^2} \int_0^\infty dk \frac{k^2 e^{-\beta(k^2 + M^2)^{\frac{1}{2}} - i\mu}}{1 - e^{-\beta(k^2 + M^2)^{\frac{1}{2}} - i\mu}}.$$

Dark matter: n might be fixed, $n = n_0$. $T \rightarrow 0$ (only ground state can contribute)

$$\left\langle : \frac{\partial T_{ij}}{\partial g_{kl}} : \right\rangle_{T=0, n=n_0} = 0,$$

Tensor structures:

$$\left\langle : \frac{\partial T_{ij}}{\partial g_{kl}} : \right\rangle_T \propto \delta_{ij} \delta^{kl} \Leftrightarrow \left. \frac{\partial T_{ij}}{\partial g_{kl}} \right|_{\text{Scalar Tensor Theory}} \propto \frac{1}{2} \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k \right)$$

$\Leftarrow$

$$\frac{\partial T_{ij}}{\partial g_{kl}} = \frac{1}{4} \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k \right) \left(\pi^2 - \sum_{n=1,2,3} (\partial_n \phi)^2 - M^2 \phi^2 \right) + \frac{1}{2} \delta_{ij} \partial^k \phi \partial^l \phi.$$

In case of thermal quanta,

1st term = 0 by on-shell condition ($E^2 - k^2 - M^2 = 0$),

2nd term $\sim \langle k^k k^l \rangle \propto \delta^{kl}$.

In case of scalar tensor theory ($M^2 \phi^2 \Rightarrow V(\phi)$),

$\phi = \phi(t) \Rightarrow$ 2nd term = 0.

When the number density of the particles is fixed $n = n_0$ and $T \rightarrow 0$ ($\Leftrightarrow$ Cold Dark Matter (CDM))

$$\left\langle : \frac{\partial T_{ij}}{\partial g_{kl}} : \right\rangle_{T=0, n=n_0} = \frac{\partial T_{ij}}{\partial g_{kl}} \Big|_{\text{Scalar-Tensor}} = 0,$$

(c.f. R. Flauger and S. Weinberg, Phys. Rev. D **97** (2018) no.12, 123506)
but in general,

$$\left\langle : \frac{\partial T_{ij}}{\partial g_{kl}} : \right\rangle_T \neq \frac{\partial T_{ij}}{\partial g_{kl}} \Big|_{\text{Scalar-Tensor}},$$

for example, $w = \frac{1}{3}$ (radiation).

Speed of Propagation

B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations],
“GW170817: Observation of Gravitational Waves from a Binary Neutron
Star Inspiral,”
Phys. Rev. Lett. **119** (2017) no.16, 161101
arXiv:1710.05832 [gr-qc]

Gravitational Wave from Neutron Star Merger

$$\left| \frac{c_{\text{GW}}^2}{c^2} - 1 \right| < 6 \times 10^{-15} .$$

c : propagating speed of the light

c_{GW} : the propagating speed of the gravitational wave

In case of covariant Galileon model

C. Deffayet, G. Esposito-Farese and A. Vikman, "Covariant Galileon," ,
Phys. Rev. D **79** (2009) 084003, doi:10.1103/PhysRevD.79.084003,
[arXiv:0901.1314 [hep-th]]

$$\mathcal{L} = X + G_4(X)R + G_{4,X} \left((\nabla^2 \phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi \right) ,$$

$$X = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi ,$$

$$G_4(X) = \frac{M_{\text{Pl}}^2}{2} + \frac{2c_0}{M_{\text{Pl}}} \phi + \frac{2c_4}{\Lambda_4^6} X^2 ,$$

c_4 term induces the modification of the effective metric for the gravitational wave,

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + C \partial_\mu \phi \partial_\nu \phi ,$$
$$\Rightarrow \left| \frac{c_{\text{GW}}^2}{c^2} - 1 \right| = \left| \frac{4c_4 x^2}{1 - 3c_4 x^2} \right| , \quad x = \frac{\dot{\phi}}{H M_{\text{Pl}}} .$$

J. Sakstein and B. Jain, "Implications of the Neutron Star Merger GW170817 for Cosmological Scalar-Tensor Theories," arXiv:1710.05893.

Propagation of Light

$$0 = \nabla^\mu F_\mu{}^\nu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}) = \nabla^2 A^\nu - \nabla^\nu \nabla^\mu A_\mu + R^{\mu\nu} A_\mu,$$

$\Rightarrow$

$$0 = \sum_{i=1,2,3} \partial_i (\partial_i A_t - \partial_t A_i),$$

$$0 = (\partial_t + H) (\partial_i A_t - \partial_t A_i) + a^{-2} \left(\Delta A_i - \partial_i \sum_{j=1,2,3} \partial_j A_j \right),$$

by assuming a spatially flat FRW metric

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2,$$

Landau gauge:

$$0 = \nabla^\mu A_\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} A_\nu) = -\partial_t A_t + 3HA_t + a^{-2} \sum_{i=1,2,3} \partial_i A_i$$
$$\Rightarrow 0 = \nabla^2 A^\nu + R^{\mu\nu} A_\mu.$$

Assume $0 = A_t = \sum_{i=1,2,3} \partial_i A_i$

$$0 = -(\partial_t^2 + H\partial_t) A_i + a^{-2} \triangle A_i.$$

de Sitter space-time $H = H_0$, $a = e^{H_0 t}$.

Assume $A_i \propto e^{i\mathbf{k}\cdot\mathbf{x}} \times$ (the part only depending on t)

Replacing $\triangle$ by $-k^2 \equiv -\mathbf{k} \cdot \mathbf{k}$.

$s \equiv e^{-H_0 t}$

$$\Rightarrow 0 = \left(\frac{d^2}{ds^2} + \frac{k^2}{H_0^2} \right) A_i ,$$

$$\Rightarrow A_i = A_{i0} \cos \left(\frac{k}{H_0} s + \theta_0 \right) .$$

Propagation in Scalar-Tensor Theory by GW170817

Gravitational wave in de Sitter space-time realized by cosmological constant,

$$u = \frac{k}{H_0} s, \quad h_{ij} = s^{-\frac{1}{2}} l_{ij}.$$

$\Rightarrow$

$$0 = \left(\frac{d^2}{du^2} + \frac{1}{u} + 1 - \frac{\left(\frac{5}{2}\right)^2}{u^2} \right) l_{ij},$$

Bessel's differential equation $\Rightarrow$ Bessel functions $J_{\pm\frac{5}{2}}(u)$.

Black hole/neutron star merger $s \equiv e^{-H_0 t} \sim 1$. $\frac{k}{H_0} \gg 1$.

$$h_{ij} \sim \frac{1}{s} \cos \left(\frac{k}{H_0} s + \frac{\pm 5 + 1}{4} \pi \right).$$

$\Rightarrow c = c_{\text{GW}}.$

Power-law expansion $a(t) = \left(\frac{t}{t_0}\right)^\alpha$ in Scalar-Tensor Theory.

$$\omega(\phi) = \frac{2\alpha}{\kappa^2 t_0^2 \phi^2}, \quad V(\phi) = \frac{3\alpha^2 - \alpha}{\kappa^2 t_0^2 \phi^2}.$$

$\sim$ perfect fluid with a constant equation of state parameter w , $\alpha = \frac{2}{3(1+w)}$.

$$H = \frac{\alpha}{t}, \quad \dot{H} = -\frac{\alpha}{t^2}.$$

Black hole/neutron star merger $\Rightarrow H \sim$ a constant, $H \sim H_0$.
 $H^2 \sim \dot{H} \Rightarrow \dot{H} \sim$ a constant, $\dot{H} = H_1$

$$0 = \left(2\dot{H} + 6H^2 + H\partial_t - \partial_t^2 + \frac{\Delta}{a^2}\right) h_{ij}$$

$\Rightarrow$

$$0 = \left(\frac{d^2}{du^2} + \frac{1}{u} + 1 - \frac{\left(\frac{5}{2}\right)^2 - \frac{2H_1}{H_0^2}}{u^2}\right) l_{ij},$$

Solution $J_{\pm \frac{5}{2} \sqrt{1 - \frac{4H_1}{25H_0^2}}}(u)$.

$$h_{ij} \sim \frac{1}{s} \cos \left(\frac{k}{H_0} s + \frac{\pm 5 \sqrt{1 + \beta} + 1}{4} \pi \right), \quad \beta \equiv -\frac{4H_1}{25H_0^2},$$

The propagation of the light is not changed.

The propagation of the gravitational wave is not changed, either.

The difference is in phase,

$$\beta = -\frac{4}{25\alpha} = -\frac{6(1+w)}{25},$$

Propagation in $F(R)$ Gravity by GW170817

$$S_{F(R)} = \int d^4x \sqrt{-g} F(R).$$

$F(R)$ gravity $\Leftrightarrow$ scalar-tensor theory, under the scale transformation
(K. i. Maeda, Phys. Rev. D **39** (1989) 3159)

$$\tilde{g}_{\mu\nu} = e^{2\Phi} g_{\mu\nu},$$

Because $\tilde{h}_i^j = \tilde{g}^{lj} \delta \tilde{g}_{il} = e^{-2\Phi} g^{lj} e^{2\Phi} \delta g_{il} = h_i^j$, h_i^j results scale invariant in this sense.

Transverse and traceless gauge,

$$\nabla^\mu h_{\mu\nu} = g^{\mu\nu} h_{\mu\nu} = 0$$

Scale transformation

$$\begin{aligned} \tilde{\nabla}^\mu \tilde{h}_\mu^\nu &= e^{-\Phi} \nabla^\mu h_{\mu\nu} + D e^{-\Phi} g^{\mu\sigma} g^{\nu\rho} \Phi_{,\sigma} h_{\mu\rho} - e^{-\Phi} g^{\nu\rho} \Phi_{,\rho} g^{\mu\sigma} h_{\mu\sigma}, \\ \Rightarrow \tilde{\nabla}^\mu \tilde{h}_\mu^\nu &= D e^{-\Phi} g^{\mu\sigma} g^{\nu\rho} \Phi_{,\sigma} h_{\mu\rho}. \end{aligned}$$

D : the dimensions of space-time.

$$\tilde{\nabla}^\mu \tilde{h}_\mu{}^\nu = D e^{-\Phi} \Phi_{,\mu} h^{\mu\nu}.$$

We assume that the background metric and therefore Φ only depend on the cosmological time t and also $g_{ti} = 0$.

$\Rightarrow$ when the perturbation with $h_{t\mu} = 0$,

$$\tilde{\nabla}^\mu \tilde{h}_\mu{}^\nu = \tilde{g}^{\mu\nu} \tilde{h}_{\mu\nu} = 0.$$

The gauge conditions for the graviton are not changed by the scale transformation.

Power law case,

$$F(R) \sim R^m \Rightarrow a(t) = \left(\frac{t}{t_0}\right)^\alpha \left(\alpha = -\frac{(m-1)(2m-1)}{m-2}\right)$$

Scale transformation $F(R)$ gravity $\Rightarrow$ Scalar-Tensor Theory.

$$\tilde{\alpha}_{\text{Scalar-Tensor}} = 3 \frac{(m-1)^2}{(m-2)^2}, \quad 1 + \tilde{w}_{\text{Scalar-Tensor}} = \frac{2}{3\tilde{\alpha}} = \frac{2(m-2)^2}{9(m-1)^2}.$$

Speed of the propagation in the gravitational wave could not be changed by the scale transformation but there is a change of the phase

$$\beta = -\frac{4}{25\tilde{\alpha}} = \frac{12(m-2)^2}{25(m-1)^2} \sim \frac{243(1+w)^2}{25},$$

(by assuming $w \sim -1$)

which is different from the case of the scalar-tensor theory.

$$\beta|_{\text{Scalar-Tensor}} = -\frac{6(1+w)}{25},$$

Gravitational Wave from Early Universe or in Future?

S. Capozziello, M. De Laurentis, S. Nojiri and S. D. Odintsov, “Evolution of gravitons in accelerating cosmologies: The case of extended gravity,” Phys. Rev. D **95** (2017) no.8, 083524, doi:10.1103/PhysRevD.95.083524, arXiv:1702.05517 [gr-qc]

Scalar Tensor Theory

Equation for Gravitational Wave

$$0 = \left(2\dot{H} + 6H^2 + H\partial_t - \partial_t^2 + \frac{\Delta}{a^2} \right) h_{ij}.$$

Assuming

$$h_{ij}(\mathbf{x}, t) = e^{i\mathbf{k}\cdot\mathbf{x}} a(t)^2 \hat{h}_{ij}(t),$$

and defining new time coordinate τ by $d\tau = a(t)^{-3} dt$,

$$0 = \frac{d^2 \hat{h}_{ij}}{d\tau^2} + 4a^6 H^2 \hat{h}_{ij} + k^2 a^4 \hat{h}_{ij}, \quad (k^2 \equiv \mathbf{k} \cdot \mathbf{k}).$$

We consider

$$a(t) = \left(\frac{t}{t_0} \right)^\alpha.$$

t_0, α : real constants.

Accelerating expansion: $\alpha \geq 1$,

Decelerating expansion: $0 < \alpha < 1$,

Phantom: $\alpha < 0$.

① When $t \rightarrow 0$,

① $\alpha > 1$. The solution oscillates $\hat{h}_{ij} \propto e^{\pm i\left(\frac{1-3\alpha}{1-\alpha}\right)k\left(\frac{t}{t_0}\right)^{1-\alpha}}$ and the absolute value of $\hat{h}_{ij}$ is finite.

② $\frac{1}{3} < \alpha < 1$. The solution diverges as $\hat{h}_{ij} \sim \left(\frac{t}{t_0}\right)^{\eta(1-3\alpha)}$. Here

$$\eta = \frac{1 \pm \sqrt{1 - \frac{16\alpha^2}{(1-3\alpha)^2}}}{2} \text{ and the real part of } \eta \text{ is positive.}$$

③ $0 < \alpha < \frac{1}{3}$ case. $\hat{h}_{ij}$ goes to a finite value $\hat{h}_{ij} \propto e^{-\frac{(1-3\alpha)^2 k^2}{2(1-\alpha)(1+\alpha)}\left(\frac{t}{t_0}\right)^{2(1-\alpha)}}$.

② When $t \rightarrow +\infty$,

① $\alpha > 1$. The solution is given by $\hat{h}_{ij} \sim \left(\frac{t}{t_0}\right)^{\eta(1-3\alpha)}$, which decreases for large t because the real part of η is positive.

② $0 < \alpha < 1$. The solution oscillates as $\hat{h}_{ij} \propto e^{\pm i\left(\frac{1-3\alpha}{1-\alpha}\right)k\left(\frac{t}{t_0}\right)^{1-\alpha}}$ and the absolute value of $\hat{h}_{ij}$ is finite.

$$F(R) \sim R^m \Rightarrow a(t) = \left(\frac{t}{t_0}\right)^\alpha, \quad \left(\alpha = -\frac{(m-1)(2m-1)}{m-2}\right)$$

When $t \rightarrow 0$,

- ① $m < \frac{1-\sqrt{3}}{2}$ ($\alpha > 1$): The solution $\hat{h}_{ij}$ oscillates but its absolute value is finite.
- ② $\frac{1-\sqrt{3}}{2} < m < \frac{1}{2}$ ($0 < \alpha < 1$): The solution diverges.
- ③ $\frac{1}{2} < m < \frac{5}{4}$ ($-5 + 2\sqrt{6} \leq \alpha < \frac{1}{3}$): The solution $\hat{h}_{ij}$ goes to a finite value.
- ④ $\frac{5}{4} < m < \frac{1+\sqrt{3}}{2}$ ($\frac{1}{3} < \alpha < 1$): The solution diverges.
- ⑤ $\frac{1+\sqrt{3}}{2} < m < 2$ ($\alpha > 1$): The solution $\hat{h}_{ij}$ oscillates but its absolute value is finite.
- ⑥ $m > 2$ ($\alpha \leq -5 - 2\sqrt{6}$): We note that this case corresponds to $t_E \rightarrow \infty$. Then the solution decreases for small t .

When $t \rightarrow +\infty$,

- ① $m < \frac{1-\sqrt{3}}{2}$ ($\alpha > 1$): The solution decreases for large t .
- ② $\frac{1-\sqrt{3}}{2} < m < \frac{1}{2}$ ($0 < \alpha < 1$): The solution oscillates and the absolute value of $\hat{h}_{ij}$ is finite.
- ③ $\frac{1}{2} < m < \frac{5}{4}$ ($-5 + 2\sqrt{6} \leq \alpha < \frac{1}{3}$): The solution oscillates and the absolute value of $\hat{h}_{ij}$ is finite.
- ④ $\frac{5}{4} < m < \frac{1+\sqrt{3}}{2}$ ($\frac{1}{3} < \alpha < 1$): The solution oscillates and the absolute value of $\hat{h}_{ij}$ is finite.
- ⑤ $\frac{1+\sqrt{3}}{2} < m < 2$ ($\alpha > 1$): The solution decreases for large t .
- ⑥ $m > 2$ ($\alpha \leq -5 - 2\sqrt{6}$): The solution oscillates but the absolute value of $\hat{h}_{ij}$ is finite.

Some differences,

For $m > 2$ or $\frac{1}{2} < m < 1$, phantom evolution, which does not appear in case of scalar tensor theory.

$t \rightarrow \infty$ corresponds to the infinite past

$t \rightarrow 0$ corresponds to the Big Rip singularity.

Near the Big Rip singularity, in case $m > 2$, $\hat{h}_{ij}$ decreases; for $\frac{1}{2} < m < 1$, $\hat{h}_{ij}$ is finite and not oscillating near the Big Rip singularity for $t \rightarrow 0$.

Even in quintessence evolution, there is a difference in the exponent.

Scalar tensor: $\hat{h}_{ij} \sim \left(\frac{t}{t_0}\right)^{\eta(1-3\alpha)}$, $F(R)$ gravity: $\hat{h}_{ij} \sim \left(\frac{t}{t_0}\right)^{\tilde{\eta}(1-3\tilde{\alpha})(-m+2)}$

$$\tilde{\alpha} \equiv 3 \frac{(m-1)^2}{(m-2)^2} \quad \tilde{\eta} = \tilde{\eta}_{\pm} \equiv \frac{1 \pm \sqrt{1 - \frac{16\tilde{\alpha}^2}{(1-3\tilde{\alpha})^2}}}{2}.$$

For example, when $m \rightarrow -\infty$ ($\alpha \rightarrow -m \rightarrow +\infty$ and $\tilde{\alpha} \rightarrow 3$)

$$\tilde{\eta}(1-3\tilde{\alpha})(-m+2) \rightarrow -\left(4 \pm 2i\sqrt{5}\right)\alpha, \quad \eta(1-3\alpha) \sim -\left(\frac{3}{2} \pm i\frac{\sqrt{7}}{2}\right)\alpha.$$

The decreasing exponent in the $F(R)$ gravity is much larger than the corresponding exponent in the scalar-tensor theory, the ratio is $8/3$.

If one is capable of fixing the exponent by observations, we can distinguish the $F(R)$ gravity from the scalar-tensor theory.

Summary

- We discussed the evolution of cosmological gravitational wave showing how the cosmological background affects their dynamics.
- The detection of cosmological gravitational wave could constitute an extremely important signature to discriminate among different cosmological models.
- We especially considered the cases of scalar-tensor gravity and $F(R)$ gravity where it is demonstrated the amplification of graviton amplitude changes if compared with General Relativity.
- We also show the speed of the gravitational wave by the modified gravity does not change different from the scalar tensor theory without higher derivative couplings.

- K. Bamba, S. Nojiri and S. D. Odintsov,
“Propagation of gravitational waves in strong magnetic fields,”
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doi:10.1103/PhysRevD.98.024002
arXiv:1804.02275 [gr-qc].

Enhancement of gravitational wave by strong magnetic fields

- NGC 6946 size ~ 100 k light years, $B \sim \mu\text{G}$
- Magnetar (e.g. SGR 1806-20) very strong magnetic field 10^{11} T.
- I. Brevik, S. Nojiri and S. D. Odintsov, in preparation.

Propagation in viscous fluid

bulk viscosity: no strong effect, shear viscosity $\sim$ friction

If the inflation is generated by the viscous fluid, there might be no primordial gravitational wave.

Thank you!

